

# INTERNATIONAL INDIAN SCHOOL BURAIDAH

TERM EXAMINATION (2019 – 20)

SUBJECT: MATHEMATICS

SET: A

CLASS: XII

Duration: 3Hours

Max. Mark: 80

## General Instructions:

1. All questions are compulsory.
2. This question paper consist of questions, divided into four sections, a, B, C and D. Section A comprises of 20 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 6 questions of 4 marks each and Section D comprises of 4 questions of 6 marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per each requirement of the question.
4. Use of calculator is not permitted.

## SECTION A

(Questions number 1 to 20 carry 1 mark each)

1. Differentiate ;  $\cos(ax + b\sin x)$  w. r. to  $x$
2. Let  $*$  be defined on  $\mathbb{N}$  by  $a*b = \frac{a+b}{2}$ . Is  $*$  a binary operation?
3. If  $y = \sin^5(2x + 1)$ ; find  $\frac{dy}{dx}$
4. A box contains 5 white, 4 red and 6 black balls. A ball is drawn at random from the box. Find the probability that the ball drawn is neither red nor black.
5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  and,  $g: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x^2$  and  $g(x) = x + 5$ . Find  $f \circ g(2)$ .
6. Differentiate ,  $8^{x^3+3x}$  w. r. to  $x$
7. Find the maximum and minimum value of,  $f(x) = \sin(2x) + 5$ .

8. Two balls are drawn at random with replacement from a box containing 12 black and 10 red balls. Find the probability that one of them is black and other is red.
9. If  $y = e^{\tan(x^5)}$  ; find  $\frac{dy}{dx}$ .
10. What is the principle value branch of ,  $\operatorname{cosec}^{-1}x$  .
11. The binary operation  $*$  :  $Z \times Z \rightarrow Z$  is defined as  $a*b = a+3b^2$ . Find  $(2*3)*4$ .
12. Differentiate,  $(x + 3)^2 \cdot (x + 4)^3 \cdot (x + 5)^4$  w.r.to  $x$  .
13. A coin is tossed , then a die is rolled . Find the probability of getting 5 , given that tail comes up.
14. Find the slope of the normal of the curve,  $x = a\cos^3\theta$  ,  $y = a\sin^3\theta$ , at  $\theta = \frac{\pi}{4}$
15. Differentiate ,  $\log(\operatorname{cose}^x)$  w.r.to  $x$  .
16. A balloon, which always remains spherical, has a variable radius. Find the rate at which the volume is increasing with respect to its radius when the radius is 10cm.
17. Find the principle value of ,  $\cos^{-1}\left(-\frac{1}{2}\right)$  .
18. Find the probability of obtaining an even prime number on each die, when a pair of dice is rolled.
19. Find the approximate change in the volume of a cube of side  $x$  meters caused by increasing the side by 2% .
20. Let  $*$  on  $Z$  defined by  $a*b = a + b - 2$ , find the identity element of  $*$  .

### SECTION B

**(Questions number 21 to 26 carry 2 marks each )**

21. If ,  $\sin^2y + \cos(xy) = \pi$ , then find  $\frac{dy}{dx}$ .

22. If  $x = a(\cos\theta + \theta\sin\theta)$ ;  $y = a(\sin\theta - \theta\cos\theta)$ , find  $\frac{dy}{dx}$

**OR**

Verify Mean value theorem for the function  $f(x) = x^2$  in  $[2, 4]$ .

23. Find the local maximum and local minimum values of  $f(x) = \frac{1}{x^2+2}$ .

24. Let  $*$  on  $Q$  defined by,  $a * b = \frac{ab}{2}$ , find the inverse of the elements.

25. Find the intervals in which the function  $f(x) = x^3 + \frac{1}{x^3}$ ,  $x \neq 0$ , is

(i) increasing (ii) decreasing

26. A and B are two events such that  $P(A) = \frac{1}{2}$ ,  $P(A \cup B) = \frac{3}{5}$  and  $P(B) = p$ , find  $p$ , if they are independent. **OR**

An experiment succeeds twice as often as it fails. Find the probability that in the next seven trials, there will be atleast 2 successes.

### SECTION C

(Questions number 27 to 32 carry 4 marks each)

27. If  $(\cos x)^y = (\cos y)^x$ , then find  $\frac{dy}{dx}$

28. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

29. Check whether the relation  $R$  in the set  $A = \{1, 2, 3, 4, 5\}$  given by

$R = \{(a, b) : |a - b| \text{ is even}\}$ , is an equivalence relation. **OR**

Let  $R$  be a relation on the set  $A$  of ordered pairs of positive integers defined by  $(x, y)R(u, v)$  if and only if  $xv = yu$ . Check whether  $R$  is an equivalence relation.

30. Check whether  $f(x) = |x-1|$  is continuous and differentiable at  $x = 1$ .

31. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

32. Find the points on the curve  $\frac{x^2}{4} + \frac{y^2}{25} = 1$ , at which the tangents are

(i) parallel to the x-axis (ii) parallel to the y-axis . **OR**

Find the equation of tangent line to the curve  $y = x^2 - 2x + 7$ , which is

(i) parallel to the line  $2x - y + 9 = 0$  (ii) perpendicular to the line  $5y - 15x = 13$ .

### SECTION D

(Questions number 33 to 36 carry 6 marks each)

33. Let  $f: R - \{-\frac{4}{3}\} \rightarrow R$  be a function defined as  $f(x) = \frac{4x}{3x+4}$ . Show that

$f$  is invertible. Find the inverse of  $f$ .

34. If  $y = e^{a \cos^{-1} x}$ , then show that  $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$ . **OR**

Differentiate,  $\cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$  w.r. to  $\tan^{-1} \left( \frac{2x}{1-x^2} \right)$ .

35. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the mean and variance of the number of queens.

36. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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